

Definite Integral Notation:

If the function $f(x)$ is continuous at every point on the interval $[a, b]$ and $F(x)$ is any anti-derivative of $f(x)$ on $[a, b]$,

then $\int_a^b f(x)dx$ is called the **definite integral** and is equal to $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

Example: Evaluate $\int_2^4 \left(x + \frac{1}{x}\right)^2 dx$

$$\begin{aligned} \int_2^4 \left(x + \frac{1}{x}\right)^2 dx &= \int_2^4 \left(x^2 + 2 + \frac{1}{x^2}\right) dx = \left[\frac{1}{3}x^3 + 2x - \frac{1}{x}\right]_2^4 \\ &= \left(\frac{1}{3}(4)^3 + 2(4) - \frac{1}{4}\right) - \left(\frac{1}{3}(2)^3 + 2(2) - \frac{1}{2}\right) \\ &= \left(\frac{64}{3} + 8 - \frac{1}{4}\right) - \left(\frac{8}{3} + 4 - \frac{1}{2}\right) \\ &= \frac{275}{12} \quad (\approx 22.92) \end{aligned}$$

Show all your details work on separate sheet(s) (8½ x 11):

1. Evaluate the following:

(a) $\int_1^4 x dx$ (b) $\int_4^9 \sqrt{x} dx$ (c) $\int_2^3 \frac{2}{x^3} dx$ (d) $\int_{16}^9 \frac{4}{\sqrt{x}} dx$

2. Evaluate the following definite integrals (giving exact answers)

(a) $\int_1^2 \left(x^2 - \frac{3}{x^4}\right) dx$ (b) $\int_0^2 (x\sqrt{x} - x) dx$ (c) $\int_0^2 (1 + 2x - 3x^2) dx$

(d) $\int_{-2}^0 (x + 1) dx$ (e) $\int_0^{-1} x^3(x + 1) dx$ (f) $\int_{-1}^1 (x + 1)(x^2 - 1) dx$

(g) $\int_1^4 (\sqrt{x} - 1)^2 dx$ (h) $\int_1^2 \left(x - \frac{1}{x}\right)^2 dx$ (i) $\int_1^3 \left(\frac{x^3 - x^2 + x}{x}\right) dx$

(j) $\int_{-1}^1 (x - x^3) dx$ (k) $\int_1^4 \frac{x+1}{\sqrt{x}} dx$ (l) $\int_1^4 \left(\sqrt{\frac{2}{x}} - \sqrt{\frac{x}{2}}\right) dx$

3. Find the indefinite integral of

(a) $\int_1^2 \frac{3}{x} dx$ (b) $\int_0^4 \frac{2}{x+1} dx$ (c) $\int_2^6 \frac{x+4}{x} dx$

(d) $\int_4^5 \left(x^2 + \frac{1}{x}\right)^2 dx$ (e) $\int_{-1}^0 \frac{3}{1-2x} dx$ (f) $\int_0^1 \frac{2}{x+1} dx$

(g) $\int_0^1 \frac{2}{(x+1)^3} dx$ (h) $\int_2^4 \left(\sqrt{x} - \frac{2}{\sqrt{x}}\right)^2 dx$ (i) $\int_3^4 \frac{2x+1}{2x^2-3x-2} dx$

